Title: La Tolteca--A Tropical Paradise

Brief Overview:

Students will analyze data given to determine time, distance, and schedules. Students will use this information to predict a ferry schedule and compute information regarding depth and oxygen intake for a scuba diving expedition.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate the ability to make conclusions about information regarding the ferry and making predictions about oxygen tanks and sea atmospheres.

- **Mathematics as Communication**
  Students will discuss conclusions based on a written summary from their results.

- **Mathematics as Reasoning**
  Students will write and estimate equations form the data to make reasonable predictions about the variables.

- **Mathematical Connections**
  Science-Students will use data to compute time vs. volume.

- **Computation and Estimation**
  Students will make estimated conclusions based on the given data.

- **Algebra**
  Students will use the distance formula to calculate a ferry schedule.

- **Statistics**
  Students will analyze the given data and make predictions on arrival and departure times around the island.

- **Measurement**
  Students should be familiar with distances using the metric system.

- **Functions**
  The student will be able to analyze relationships using data and graphs.

- **Trigonometry**
  Students will use trig functions to determine distance from one point to another on the island and use the graphs of sine or cosine to draw conclusions from the data.

Links to Science Standards:

- **Unifying Concepts and Processes in Science**
  Students will demonstrate concepts in sea atmospheres and use mathematical equations to predict future outcomes.

- **Science and Technology**
  Students will use TI-83 calculators to develop a linear equation.

Grade/Level:

7-12 -- General Science and with enrichment activities for Algebra II/Trigonometry

Duration/Length:

One ninety minute period or two forty minute periods should be required.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Basic use of the TI-83 (Statistics input, Stat Plot setup, and graphing)
- Use of the distance formula to find rate or time
- Familiarity with time lapse
- Reading and interpreting information on a graph and a chart
- Writing linear equations
- Familiarity with atmospheres below sea level
- Identifying and interpreting the sine and cosine graphs.(For trig extension only)

Objectives:

Students will:

- work cooperatively in groups of 2 or 3.
- find duration and times for a ferry schedule using the distance formula.
- use a TI-83 graphing calculator to input data to determine a regression equation and to make a scatter plot.
- write a linear equation to determine the relationship of an oxygen tank and air pressure.
- use the equation to make predictions on a scuba dive.
- use the trig functions to find the distance between two points and use the graphs to analyze the given information.

Materials/Resources/Printed Materials:

- TI-83 Graphing Calculator
- One activity packet per group.

Development/Procedures:

- Divide students into groups of 2 or 3.
- Read scenario to entire class and see if there are any questions.
- Students answer questions using data provided.
- Students will input data into the TI-83 to graph a scatter plot and determine the regression equation.
- For trig applications, students will use trig functions to calculate distance between two points and analyze the trig graphs.

Performance Assessment:

Teacher should assess the students on the basis of their groups results from their activity packet and whether or not the students conclusions are valid arguments. Teachers should also observe the students while they are working within their groups to make sure they are all working together on the activities.

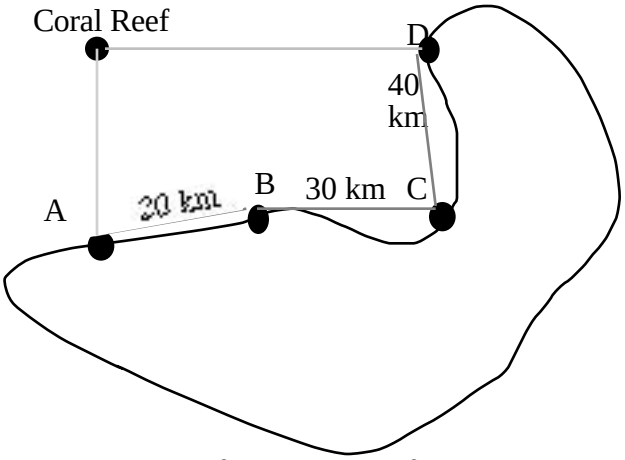
Extension/Follow Up:

- This unit may be used for trigonometry classes through the extension activity.
- Teacher may also have students calculate speed and distance of ferry including other factors such as tides or weather conditions.

Authors:

Jennie R. Burgee	Betty Raymond	Carol West
Elizabeth Seton High	James Madison Middle	Elizabeth Seton High
Archdiocese of Wash., DC	Prince George's County, MD	Archdiocese of Wash., DC

Activity #1



On the beautiful tropical island of La Tolteca, a ferry carries sightseers from one point to another, traveling from point A to point B to point C to point D then back to points C, B, and A. If the following are the distances between ports, answer the questions below:

A to B	20 km
B to C	30 km
C to D	40 km

1. If the ferry travels 50 km per hour, find the duration of each leg of the trip (in minutes)

A to B _____

B to C _____

C to D _____

2. Fill in the chart below. The ferry runs approximately 10 hours per day

Time	Port	Dist From A (Km)	Cumulative Minutes
8:00 AM	A	0	
	B		
	C		
	D		
	C		
	B		
	A	0	
	B		
	C		
	D		
	C		
	B		
	A	0	

Activity #2

The island of LaTolteca is well know for its extensive coral reefs. To take advantage of this, you decide to go scuba diving while visiting the island.

The dive club and your equipment are located at point "C" on the island. Your lodging is located at point "B". You must meet your dive group at point "A" by 11:45 a.m. to begin the dive.

Problem #1:

What time will you need to catch the ferry to arrive at point "C" to gather your gear and get to point ' A" for the dive by 11:45 a.m.? (Refer to chart from Activity #1) Explain your reasoning in determining this departure time.

Problem #2:

The capacity of your oxygen tank is 80 cubic inches of air. This equals 3,000 psi (pounds per square inch). Each atmosphere below sea level consists of 33 feet. The pressure doubles at each atmosphere and the air is compressed to 1/2 its size.

Write equations which would show the relationship between the original volume and pressure of the tank and the change in the tank upon your decent.

Problem #3:

Go to your groups and brainstorm the possible events which could occur to alter your schedule to meet your ferry and arrive on time for your dive. Make a master list of all ideas. Be creative and stretch your brain.

Chose some ideas from this list to write a humorous descriptive paragraph describing the situation(s) which altered your schedule to arrive on time for your dive.

Applications for Trig Students

Note: For questions 2, 3, students need to have experience graphing sine and cosine graphs.

1. Suppose the total straight-line distance from point A to point D is 60 km. Point D is located due NE of point A, and there is a hidden coral reef, where scuba divers explore, directly north of point A and west of point D. Let's calculate the distance from point D to the coral reef.

First make a sketch and find the angles.

Now, which trig function or formula would be appropriate?

Now answer the question-

What is the distance from point D to the coral reef?

2. Using your TI-83 calculator, enter the total times in List 1 and the distances from various ports to your stop(A) in List 2.

Use the stat plot to display a scatter plot.(see teacher notes for specific instructions)

Now calculate the regression equation. Which type fits best?

Notice that the values repeat in a cycle of values.

3. Use your equation to answer the following:

a) at 2 PM, how far will the ferry be from your position at A?

(hint x = how many minutes?)

b) Find several times of day when the ferry will arrive at point D.

TI-83 Instructions

1. Entering the data in two lists, press these keys:

   Clear the lists in memory by pressing [2nd] [+(MEM)]

   Select Option 4 and [ENTER]

   Then enter the data by pressing : [STAT]

   Select Option 1 and [ENTER]

   Enter the total cumulative time the ferry has traveled in L1, pressing [Enter] after each entry or "arrow" down.

   Use the right arrow to get to L2 and enter the distances from various ports to your stop(A) in L2 as directed above.

2. Making a scatter plot:

   press [y =] and clear any equations

   press [2nd] [y =] then select Plot 1 and [Enter]

   Highlight the following choices by placing the blinking cursor over

   ON, then  (scatter plot) and the default mark is the "box"

   Choose L1 for your x-list, L2 for your y-list

   Press [Zoom] 9, Then [Graph]

A scatter plot should then appear on your calculator screen.

Finding and Graphing a Regression Equation on the TI-83 Calculator

1. Press [Stat] then arrow to CALC

Choices 4-10 and A through C allow you to choose the type of equation you think may best fit your data.

Do all the data points appear to lie on a straight line ? _____
Do the points lie on a parabola? _____

What type of graph have we studied which may fit this pattern of data points?
_____

Press the number or letter of your choice and the equation will be computed for you. When you make your choice, remember to name L1, L2 then [Enter]

2. To graph this equation on the same screen with the data points, and check the fit

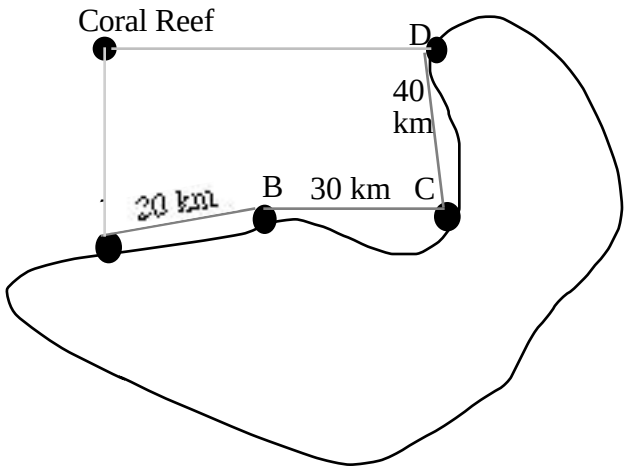
press [y =] then CLEAR ANY CURRENT FUNCTION FROM y1
then press [Vars] SELECT #5 (statistics)

move the cursor to the right TWICE (EQ) and press 1 (RegEq)

and the equation will be "pasted" in the $y_1 =$

3. Press [Graph] and OBSERVE how well this graph passes through the data points!!

Activity #1 Answer Key



On the beautiful tropical island of La Tolteca, a ferry carries sightseers from one point to another, traveling from point A to point B to point C to point D then back to points C, B, and A. If the following are the distances between ports, answer the questions below:

A to B	20 km
B to C	30 km
C to D	40 km

1. If the ferry travels 50 km per hour, find the duration of each leg of the trip (in minutes)

Use D = R x T

A to B **0.4 hr = 24 min**

B to C **0.6 hr = 36 min**

C to D **0.8 hr = 48 min**

2. Fill in the chart below. The ferry runs approximately 10 hours per day

Time	Port	Dist From A (km)	Cumulative Minutes
8:00 AM	**A**	**0**	**0**
	B	**20**	**24**
	C	**50**	**24 + 36 = 60**
	D	**90**	**60 + 48 = 108**
	C	**50**	**108 + 48 = 156**
	B	**20**	**156 + 36 = 192**
	A	**0**	**192 + 24 = 216**
	B	**20**	**216 + 24 = 240**
	C	**50**	**240 + 36 = 276**
	D	**90**	**276 + 48 = 324**
	C	**50**	**324 + 48 = 372**
	B	**20**	**372 + 36 = 408**
	A	**0**	**408 + 24 = 432 = 7.2 hr.**

Activity #2 Answer Key

The island of LaTolteca is well know for its extensive coral reefs. To take advantage of this, you decide to go scuba diving while visiting the island.

The dive club and your equipment are located at point "C" on the island. Your lodging is located at point "B". You must meet your dive group at point "A" by 11:45 a.m. to begin the dive.

Problem #1:

What time will you need to catch the ferry to arrive at point "C" to gather your gear and get to point ' A" for the dive by 11:45 a.m.? (Refer to chart from Activity #1) Explain your reasoning in determining this departure time. **You must leave at 8:33 or before**

B	C	D	C	B	A
8:33	9:09	9:57	10:45	11:21	11:45

Problem #2:

The capacity of your oxygen tank is 80 cubic inches of air. This equals 3,000 psi (pounds per square inch). Each atmosphere below sea level consists of 33 feet. The pressure doubles at each atmosphere and the air is compressed to 1/2 its size.

Write equations which would show the relationship between the original volume and pressure of the tank and the change in the tank upon your decent.

n = # of atmospheres

$V_n = V_{n-1} / 2$

$P_n = P_{n-1} (2)$

$V_n = V_1 (1/2)^n$

$P_n = P_1 (2)^n$

Need worksheet to see a pattern

# of atmos	Volume	Pressure
0	80	3000
1	40	6000
2	20	12000
3	10	24000
4		
5		

Problem #3:

Go to your groups and brainstorm the possible events which could occur to alter your schedule to meet your ferry and arrive on time for your dive. Make a master list of all ideas. Be creative and stretch your brain.

Weather, breakdown (variable)

Chose some ideas from this list to write a humorous descriptive paragraph describing the situation(s) which altered your schedule to arrive on time for your dive.

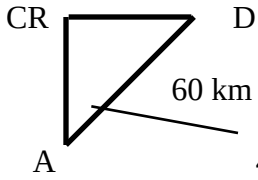
Variable

Applications for Trig Students Answer Key

Note: For questions 2, 3, students need to have experience graphing sine and cosine graphs.

1. Suppose the total straight-line distance from point A to point D is 60 km. Point D is located due NE of point A, and there is a hidden coral reef, where scuba divers explore, directly north of point A and west of point D. Let's calculate the distance from point D to the coral reef.

First make a sketch and find the angles.



D to CR = 60 / $\sqrt{2}$ = 42.43 km

Now, which trig function or formula would be appropriate?

45o - 45o ratio of sides leg = hypotenuse / square root 2

Now answer the question-

What is the distance from point D to the coral reef?

60 / $\sqrt{2}$ = 42.43 km

2. Using your TI-83 calculator, enter the total times in List 1 and the distances from various ports to your stop(A) in List 2.

Use the stat plot to display a scatter plot.(see teacher notes for specific instructions)

Now calculate the regression equation. Which type fits best?

Trig Regression, y = 40.75 + sin (.0298x + -1.6225) + 44.86
(need graphing calculator)

Notice that the values repeat in a cycle of values.

3. Use your equation to answer the following:

a) at 2 PM, how far will the ferry be from your position at A?

(hint x = how many minutes?) **85.77 km (need graphing calculator)**

b) Find several times of day when the ferry will arrive at point D.

when x = 90 Answers will vary.

Answer Key

Finding and Graphing a Regression Equation on the TI-83 Calculator

1. Press [Stat] then arrow to CALC

Choices 4-10 and A through C allow you to choose the type of equation you think may best fit your data.

Do all the data points appear to lie on a straight line ? ____**No**___
Do the points lie on a parabola? ____**No** __

What type of graph have we studied which may fit this pattern of data points?
trig graph - has repeated values (cycles)

Press the number or letter of your choice and the equation will be computed for you. When you make your choice, remember to name L1, L2 then [Enter]

2. To graph this equation on the same screen with the data points, and check the fit

press [y =] then CLEAR ANY CURRENT FUNCTION FROM y1
then press [Vars], SELECT #5 (statistics)

move the cursor to the right TWICE (EQ) and press 1 (RegEq)

and the equation will be "pasted" in the $y_1 =$

3. Press [Graph] and OBSERVE how well this graph passes through the data points!!

Scoring Rubric

Activity #1

Pts	
3	Student was successfully able to calculate the time it took the ferry to travel from one point to the other using the distance formula. Student also was able to calculate the distance from one point to the next and calculate cumulative minutes of the ferry trip.
2	Student was successfully able to calculate the duration of each leg of the trip in minutes . Student also had some difficulty calculating the distance from one point to the next or calculating cumulative minutes of the ferry trip.
1	Student had difficulty calculating the duration of each leg of the trip in minutes . Student also had difficulty calculating the distance of the ferry from point A or calculating cumulative minutes of the ferry trip.
0	Student had little understanding of how to calculate the duration of the ferry trip. Student also had no understanding of how to calculate distance of the ferry and how to calculate cumulative minutes. Student may have left most or all of the activity blank.

Scoring Rubric

Activity #2

Pts	
3	Student had an understanding of how to calculate the departure time for the trip. Student also was able to write an equation to illustrate the relationship between the original volume and pressure of the oxygen tank. Student was able to create ideas that could alter their diving plans and was able to communicate these ideas in a descriptive and well-written paragraph.
2	Student was successfully able to calculate the the departure time for the trip. Student had some difficulty writing an equation that would show the relationship between the original volume and pressure of the oxygen tank. Student was able to come up with a creative list of ideas that could alter their diving plans and was able to communicate these ideas in a good written paragraph.
1	Student had difficulty calculating the departure time for the trip. Student also was not able to write an equation showing the relationship between the volume and pressure of the tank and the changes in the tank. Student developed few ideas on what could change the schedule and had difficulty writing these ideas clearly in a well-written paragraph.
0	Student had little understanding of how to calculate the departure time for the trip. Student also was not able to write an equation showing the relationship between the volume and pressure of the tank and the changes in the tank. Student had no ideas on what could change the schedule. The activity may be incomplete or blank altogether.

Scoring Rubric

Trig Students

Pts	
3	Student was successfully able to use the trig formulas to calculate the distance from point D to the coral reef. Student also was able to find the regression equation and find the best fit using the TI-83. Student was able to use the equation to calculate the position of the ferry at various times.
2	Student was successfully able to calculate the the distance from point D to the coral reef. Student also had some difficulty in calculating the regression equation or using the equation to calculate the position of the ferry at various times.
1	Student was able to calculate the distance from point D to the coral reef. Student also had lot of difficulty in calculating the regression equation and using the equation to calculate the position of the ferry at various times.
0	Student had difficulty in calculating the distance from point D to the coral reef. Student also was not able to calculate the regression equation and use the equation to calculate the position of the ferry at various times. Student may have left assignment blank or incomplete.